DAY THIRTY TWO

Three Dimensional Geometry

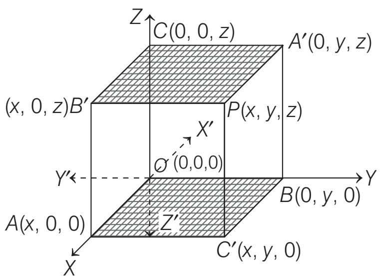
Learning & Revision for the Day

- Coordinates of a Point in a Space
- Section Formula
- Direction Cosines and Ratios
- Equation of Line in Space
- Skew-Lines
- Coplanar Lines
- Plane
- Angle between a Line and a Plane

Coordinates of a Point in a Space

From the adjoining figure, we have

- The three mutually perpendicular lines in a space which divides the space into eight parts are called coordinates axes.
- The coordinates of a point are the distances from the origin to the feet of the perpendiculars from the point on the respective coordinate axes.
- The coordinates of any point on the X, Y and Z-axes will be as $(x, 0, 0)$ $(0, y, 0)$ and $(0, 0, z)$ respectively and the coordinates of any point P in space will be as (x, y, z).



Distance between Two Points

The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Section Formula

If $M(x, y, z)$ divides the line joining of points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio $m : n$, then

For Internal Division

$$x = \frac{mx_2 + nx_1}{m+n},\ y = \frac{my_2 + ny_1}{m+n} \text{ and } z = \frac{mz_2 + nz_1}{m+n}$$

For External Division

$$x = \frac{mx_2 - nx_1}{m-n},\ y = \frac{my_2 - ny_1}{m-n} \text{ and } z = \frac{mz_2 - nz_1}{m-n}$$

The coordinates of the mid-point of the line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Some Important Results

1. If $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are the vertices of a ΔABC, then

   (i) Centroid of triangle $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

   (ii) Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$

   (iii) If area of $\Delta ABC = 0$, then these points are collinear.

2. Four non-coplanar points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ form a tetrahedron with vertices A, B, C and D, edges AB, AC, AD, BC, BD and CD, faces ABC, ABD, ACD and BCD, then

   (i) Centroid $\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$

   (ii) Volume $= \frac{1}{6}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix}$

Direction Cosines and Ratios

If a vector makes angles α, β and γ with the positive directions of X-axis, Y-axis and Z-axis respectively, then $\cos\alpha, \cos\beta$ and $\cos\gamma$ are called its **direction cosines** and they are denoted by l, m, n, i.e. $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$.

If numbers a, b and c are proportional to l, m and n respectively, then a, b and c are called **direction ratios.**

Thus, a, b and c are the direction ratios of a vector, provided $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

Important Results

- A line in space can be extended in two opposite directions and so it has two sets of direction cosines.
- In order to get unique set of direction cosines, we must take the given line as a directed line.
- Let L is a directed line which makes α, β and γ with positive direction of X, Y and Z-axis, respectively. If we reverse the direction of L, then direction angles are replaced by their supplements, i.e. $\pi - \alpha, \pi - \beta, \pi - \gamma$.
- If the line does not pass through origin, then draw a line through origin and parallel to given line and then find its direction cosines as two parallel lines have same set of direction cosines.

Some Important Deductions

(i) Direction ratios of the line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$ and its direction cosines are $\frac{x_2 - x_1}{|\mathbf{PQ}|}, \frac{y_2 - y_1}{|\mathbf{PQ}|}, \frac{z_2 - z_1}{|\mathbf{PQ}|}$.

(ii) If $P(x, y, z)$ is a point in space and $OP = \mathbf{r}$ then

(a) $x = l|\mathbf{r}|, y = m|\mathbf{r}|, z = n|\mathbf{r}|$

(b) $l|\mathbf{r}|, m|\mathbf{r}|$ and $n|\mathbf{r}|$ are projections of $\mathbf{r}$ on OX, OY and OZ, respectively.

(c) $\mathbf{r} = |\mathbf{r}|(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$ and $\hat{\mathbf{r}} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$

(d) If $\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, then a, b, c are DR's of vector and DC's are given by $l = \frac{a}{|\mathbf{r}|}, m = \frac{b}{|\mathbf{r}|}, n = \frac{c}{|\mathbf{r}|}$

(iii) The sum of squares of direction cosines is always unity, i.e. $l^2 + m^2 + n^2 = 1$

(iv) Direction cosines are unique but direction ratio are not unique and it can be infinite.

(v) If a, b, c are DR's of a line and l, m, n are DC's of a line, then $l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$, $m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$ and $n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$

(vi) The DC's of a line which is equally inclined to the coordinate axes are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$.

(vii) If l, m and n are the DC's of a line, then the maximum value of $lmn = \frac{1}{3\sqrt{3}}$.

Equations of a Line in Space

Equation of line passing through point $A(\mathbf{a})$ and parallel to vector $(\mathbf{b})$ is $\mathbf{r} = \mathbf{a} + \lambda\,\mathbf{b}$.

If coordinates of A be (x_1, y_1, z_1) and the direction ratios of line be a, b and c, then equation of line is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$.

NOTE
- Equation of X-axis is $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$ or $y = 0, z = 0$
- Equation of Y-axis is $\frac{x-0}{0} = \frac{y-0}{1} = \frac{z-0}{0}$ or $x = 0, z = 0$
- Equation of Z-axis is $\frac{x-0}{0} = \frac{y-0}{0} = \frac{z-0}{1}$ or $x = 0, y = 0$

Equation of a line passing through two given points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

Its vector form is $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$.

The parametric equations of a line through (a_1, a_2, a_3) with DC's l, m and n are $x = a_1 + lr$, $y = a_2 + mr$ and $z = a_3 + nr$.

Angle between Two Intersecting Lines

1. If DR's of two lines are a_1, b_1, c_1 and a_2, b_2, c_2, then
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}}$$
   (i) Condition for parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
   (ii) Condition for perpendicular lines,
$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
2. Angle between two lines with DC's l_1, m_1, n_1 and l_2, m_2, n_2 is $\cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$
   or $\sin^{-1}\left(\sqrt{\Sigma(m_1n_2 - m_2n_1)^2}\right)$.
   (i) Condition for parallel lines, $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$
   (ii) Condition for perpendicular lines,
   $l_1l_2 + m_1m_2 + n_1n_2 = 0$

NOTE
- The angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.
- The angle between a diagonal of a cube and a face is $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$.
- The angle between the diagonal of a cube and edge of cube is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.
- If a straight line makes angles α, β, γ and δ with the diagonals of a cube, then
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$

Skew-Lines

Two straight lines in a space which are neither parallel nor intersecting are called skew-lines. Thus, skew-lines are those lines which do not lie in the same plane.

Shortest Distance between Two Skew-Lines

- If l_1 and l_2 are two skew-lines, then there is one and only one line perpendicular to each of the line l_1 and l_2, which is known as the line of shortest distance.
- The shortest distance between two lines l_1 and l_2 is the distance PQ between the points P and Q, where the line of shortest distance intersects the two given lines.
- The shortest distance between two skew-lines $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + \mu\,\mathbf{d}$ is given by SD $= \left|\frac{(\mathbf{c} - \mathbf{a})\cdot(\mathbf{b} \times \mathbf{d})}{|\mathbf{b} \times \mathbf{d}|}\right|$
- The shortest distance between the lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is
$$\text{SD} = \left|\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum (b_1c_2 - b_2c_1)^2}}\right|$$
- Two lines $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + \mu\mathbf{d}$ are intersecting if shortest distance between them is zero.
  i.e. $\left|\frac{(\mathbf{c} - \mathbf{a})\cdot(\mathbf{b} \times \mathbf{d})}{|\mathbf{b} \times \mathbf{d}|}\right| = 0 \Rightarrow (\mathbf{c} - \mathbf{a})\cdot(\mathbf{b} \times \mathbf{d}) = 0$
  or $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

Distance or shortest distance between two parallel lines

- Shortest distance between parallel lines will be the perpendicular distance.
- If the parallel lines are given by $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + \mu\mathbf{b}$ then distance between them is $d = \left|\frac{|(\mathbf{c} - \mathbf{a}) \times \mathbf{b}|}{|\mathbf{b}|}\right|$

Coplanar Lines

Lines which lie in the same plane are called coplanar lines. Any two coplanar lines are either parallel or intersecting.

Condition for Coplanarity of Two non-parallel Lines

Two lines $\mathbf{r} = \mathbf{a} + \lambda\,\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + \mu\,\mathbf{d}$ are coplanar or intersecting, if $(\mathbf{c} - \mathbf{a})\cdot(\mathbf{b} \times \mathbf{d}) = 0 \Rightarrow [\mathbf{a}\ \mathbf{b}\ \mathbf{d}] = [\mathbf{c}\ \mathbf{b}\ \mathbf{d}]$

The lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$

and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar,

if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0.$

Plane

A plane is a surface such that line joining any two points of the plane totally lies in it.

Equation of a Plane in Different Forms

1. The general equation of a plane is $ax + by + cz + d = 0$ and $a^2 + b^2 + c^2 \neq 0$, where, a, b and c are the DR's of the normal to the plane.

(i) Plane through the origin is $ax + by + cz = 0$.

(ii) Planes parallel to the coordinate planes (perpendicular to coordinate axes) $x = k$ parallel to YOZ plane, $y = k$ parallel to ZOX plane and $z = k$ parallel to XOY plane.

(iii) Planes parallel to coordinate axes
$by + cz + d = 0$ parallel to X-axis
$ax + cz + d = 0$ parallel to Y-axis
$ax + by + d = 0$ parallel to Z-axis

2. If a, b and c are the intercepts of plane with the coordinate axes, then equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. It meets the coordinate axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$.

3. (i) If l, m and n are DC's of normal to the plane, p is the distance of the origin from the plane, then equation of plane is $lx + my + nz = p$.

(ii) Coordinates of foot of perpendicular, drawn from the origin to the plane, is (lp, mp, np).

(iii) If $\mathbf{ON}$ is the normal from the origin to the plane and $\hat{\mathbf{n}}$ is the unit vector along $\mathbf{ON}$. Then $\mathbf{ON} = p\hat{n}$ and equation of plane is $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = p$, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

4. Plane through a point (x_1, y_1, z_1) is
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.
where a, b, c are DR's of normal to the plane.

5. Plane through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

6. (i) Equation of plane, passing through a point A with position vector $\mathbf{a}$ and is parallel to given vectors $\mathbf{b}$ and $\mathbf{c}$, is $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{b} \times \mathbf{c}) = 0$ or $[\mathbf{r} - \mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0$

(ii) Its cartesian equation is $\begin{vmatrix} x - a_1 & y - a_2 & z - a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

7. Plane parallel to the given plane $ax + by + cz + d = 0$ is $ax + by + cz + k = 0$, where k is a constant determined by the given condition.

8. (i) Any plane passing through the line of intersection of the planes $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$ is $(ax + by + cz + d) + \lambda(a_1x + b_1y + c_1z + d_1) = 0$

(ii) If $\mathbf{r} \cdot \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$ are two planes, then their line of intersection is perpendicular to both $\mathbf{n}_1$ and $\mathbf{n}_2$, i.e. line is parallel to the vectors $\mathbf{n}_1 \times \mathbf{n}_2$.

Some Important Results on plane

- If $ax + by + cz + d_1 = 0$ and $a_1x + b_1y + c_1z + d_2 = 0$ are the equations of any two planes, then $ax + bx + cz + d_1 = 0$ $= a_1x + b_1y + c_1z + d_2$ gives the equation of straight line.
- Plane $ax + by + cz + d = 0$ intersecting a line segment joining $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ divides it in the ratio $-\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$
  (i) If this ratio is positive, then A and B are on opposite sides of the plane.
  (ii) If this ratio is negative, then A and B are on the same side of the plane.
- If θ be the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, then
$$\theta = \cos^{-1}\left(\left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|\right)$$
- Two planes are parallel if their normals are parallel and the planes are perpendicular if their normals are perpendicular.
- If $\mathbf{r} \cdot \mathbf{n}_1 = d_1$ (or $a_1x + b_1y + c_1z = d_1$) and $\mathbf{r} \cdot n_2 = d_2$ (or $a_2x + b_2y + c_2z = d_2$) are two planes, then they are
  (i) parallel if $\mathbf{n}_1 = \lambda\mathbf{n}_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
  (ii) perpendicular if $\mathbf{n}_1 \cdot \mathbf{n}_2 = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$.
- Distance of a point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

  Distance of the origin is $\frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$.

  The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

Angle between a Line and a Plane

If angle between the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ and the plane $a_1x + b_1y + c_1z + d = 0$ is θ, then $(90° - \theta)$ is the angle between normal and the line, therefore

$$\cos(90° - \theta) = \left|\frac{aa_1 + bb_1 + cc_1}{\sqrt{a^2 + b^2 + c^2}\sqrt{a_1^2 + b_1^2 + c_1^2}}\right|$$

- If $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$, then line is perpendicular to plane.
- If $a \cdot a_1 + b \cdot b_1 + c \cdot c_1 = 0$, then line is parallel to plane.
- If $a \cdot a_1 + b \cdot b_1 + c \cdot c_1 = 0$, and $a_1x_1 + b_1y_1 + c_1z_1 + d = 0$, then line lies in the plane.

Important Points Related to Line and Plane

- Projection of a line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) on a line with direction cosine l, m, n is $|(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n|$

- Foot of the perpendicular from a point (x_1, y_1, z_1) on the plane $ax + by + cz + d = 0$ is (x, y, z), where $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = -\frac{(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$.
- Image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is (x, y, z), where $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = -\frac{2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$.
- Four points (x_i, y_i, z_i), where $i = 1, 2, 3$ and 4 are coplanar, if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0.$
- Planes bisecting the angle between two intersecting planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are given by $\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm \frac{(a_2x + b_2y + c_2z + d_2)}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$
  - (i) If $a_1a_2 + b_1b_2 + c_1c_2 < 0$, then origin is in acute angle and the acute angle bisector is obtained by taking positive sign in the above equation. The obtuse angle bisector is obtained by taking negative sign in the above equation.
  - (ii) If $a_1a_2 + b_1b_2 + c_1c_2 > 0$, then origin lies in obtuse angle and the obtuse angle bisector is obtained by taking positive sign in above equation. Acute angle bisector is obtained by taking negative sign.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 If the orthocentre and centroid of a triangle are $(-3, 5, 1)$ and $(3, 3, -1)$ respectively, then its circumcentre is

(a) $(6, 2, -2)$ (b) $(1, 2, 0)$ (c) $(6, 2, 2)$ (d) $(6, -2, 2)$

2 A line makes the same angle θ with each of the X and Z-axes. If the angle β, which it makes with Y-axis, is such that $\sin^2 \beta = 3 \sin^2 \theta$, then $\cos^2 \theta$ is equal to → **AIEEE 2004**

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

3 A line makes an angle θ with X and Y-axes both. A possible value of θ is in

(a) $\left[0, \frac{\pi}{4}\right]$ (b) $\left[0, \frac{\pi}{2}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{3}, \frac{\pi}{6}\right]$

4 The projections of a vector on the three coordinate axes are 6, -3 and 2, respectively. The direction cosines of the vector are → **AIEEE 2009**

(a) $6, -3, 2$ (b) $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$ (c) $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$ (d) $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

5 If the projections of a line segment on the X, Y and Z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is → **JEE Mains 2013**

(a) 12 (b) 7 (c) 9 (d) 6

6 A vector $\mathbf{r}$ is inclined at equal angles to OX, OY and OZ. If the magnitude of $\mathbf{r}$ is 6 units, then $\mathbf{r}$ is equal to

(a) $\sqrt{3}\,(\mathbf{i} + \mathbf{j} + \mathbf{k})$ (b) $-\sqrt{3}\,(\mathbf{i} + \mathbf{j} + \mathbf{k})$
(c) $-2\sqrt{3}\,(\mathbf{i} + \mathbf{j} + \mathbf{k})$ (d) None of these

7 A line L_1 passes through the point $3\mathbf{i}$ and is parallel to the vector $-\mathbf{i} + \mathbf{j} + \mathbf{k}$ and another line L_2 passes through the point $\mathbf{i} + \mathbf{j}$ and is parallel to the vector $\mathbf{i} + \mathbf{k}$, then point of intersection of the lines is

(a) $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (b) $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ (c) $\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ (d) $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

8 The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses the YZ-plane at the point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$. Then, → **AIEEE 2008**

(a) $a = 8, b = 2$ (b) $a = 2, b = 8$
(c) $a = 4, b = 6$ (d) $a = 6, b = 4$

9 The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is → **AIEEE 2005**

(a) 30° (b) 45° (c) 90° (d) 0°

10 The angle between a diagonal of a cube and an edge of the cube intersecting the diagonal is

(a) $\cos^{-1} \frac{1}{3}$ (b) $\cos^{-1} \sqrt{\frac{2}{3}}$
(c) $\tan^{-1} \sqrt{2}$ (d) None of these

11 The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is → **JEE Mains 2014**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

12 If the line, $\frac{x - 3}{2} = \frac{y + 2}{-1} = \frac{z + 4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to → **JEE Mains 2016**

(a) 26 (b) 18
(c) 5 (d) 2

13 The direction cosines of two lines at right angles are $(1, 2, 3)$ and $\left(-2, \frac{1}{2}, \frac{1}{3}\right)$, then the direction cosine perpendicular to both the given lines are

(a) $\sqrt{\frac{25}{2198}}, \frac{19}{\sqrt{2198}}, \sqrt{\frac{729}{2198}}$ (b) $\sqrt{\frac{24}{2198}}, \sqrt{\frac{38}{2198}}, \sqrt{\frac{730}{2198}}$
(c) $\frac{1}{3}, -2, \frac{-7}{2}$ (d) None of these

14 The foot of perpendicular from $(0, 2, 3)$ to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, is

(a) $(-2, 3, 4)$ (b) $(2, -1, 3)$ (c) $(2, 3, -1)$ (d) $(3, 2, -1)$

15 The projection of the line segment joining $(2, 5, 6)$ and $(3, 2, 7)$ on the line with direction ratios $2, 1, -2$, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 1

16 The shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$, is → **NCERT**

(a) $\sqrt{29}$ units (b) 29 units (c) $\frac{29}{2}$ units (d) $2\sqrt{29}$ units

17 The shortest distance between the diagonals of a rectangular parallelopiped whose sides are a, b, c and the edges not meeting it, are

(a) $\frac{bc}{\sqrt{b^2 - c^2}}, \frac{ca}{\sqrt{c^2 - a^2}}, \frac{ab}{\sqrt{a^2 - b^2}}$

(b) $\frac{bc}{\sqrt{b^2 + c^2}}, \frac{ca}{\sqrt{c^2 + a^2}}, \frac{ab}{\sqrt{a^2 + b^2}}$

(c) $\frac{2bc}{\sqrt{b^2 - c^2}}, \frac{2ca}{\sqrt{c^2 - a^2}}, \frac{2ab}{\sqrt{a^2 - b^2}}$

(d) None of the above

18 If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to → **AIEEE 2012**

(a) -1 (b) $\frac{2}{9}$ (c) $\frac{9}{2}$ (d) 0

19 If the straight lines $x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$ and $x = \frac{t}{2}, y = 1 + t, z = 2 - t$, with parameters s and t respectively are coplanar, then λ is equal to → **AIEEE 2004**

(a) -2 (b) -1 (c) $-\frac{1}{2}$ (d) 0

20 The distance of the point $(1, 0, 2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$ is → **JEE Mains 2015**

(a) $2\sqrt{14}$ (b) 8 (c) $3\sqrt{21}$ (d) 13

21 A vector **n** is inclined to X-axis at 45°, to Y-axis at 60° and at an acute angle to Z-axis. If **n** is a normal to a plane passing through the point $(\sqrt{2}, -1, 1)$, then the equation of the plane is → **JEE Mains 2013**

(a) $4\sqrt{2}x + 7y + z = 2$ (b) $\sqrt{2}x + y + z = 2$
(c) $3\sqrt{2}x - 4y - 3z = 7$ (d) $\sqrt{2}x - y - z = 2$

22 Let Q be the foot of perpendicular from the origin to the plane $4x - 3y + z + 13 = 0$ and R be a point $(-1, 1, -6)$ on the plane. Then, length QR is → **JEE Mains 2013**

(a) $\sqrt{14}$ (b) $\sqrt{\frac{19}{2}}$ (c) $3\sqrt{\frac{7}{2}}$ (d) $\frac{3}{\sqrt{2}}$

23 The plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, -1, 2)$ and $(1, 1, 1)$ makes intercepts on the coordinate axes and the sum of whose length is

(a) 3 (b) 6 (c) 12 (d) 20

24 The coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane passing through three points $(2, 2, 1)$, $(3, 0, 1)$ and $(4, -1, 0)$, is → **NCERT Exemplar**

(a) $(1, 2, 7)$ (b) $(-1, 2, -7)$
(c) $(1, -2, 7)$ (d) None of these

25 The volume of the tetrahedron formed by coordinate planes and $2x + 3y + z = 6$, is

(a) 5 (b) 4 (c) 6 (d) 0

26 The equation of the plane passing through $(2, 1, 5)$ and parallel to the plane $3x - 4y + 5z = 4$ is

(a) $3x - 4y + 5z - 27 = 0$ (b) $3x - 4y + 5z + 21 = 0$
(c) $3x - 4y + 5z + 26 = 0$ (d) $3x - 4y + 5z + 17 = 0$

27 If Q is the image of the point $P(2, 3, 4)$ under the reflection in the plane $x - 2y + 5z = 6$, then the equation of the line PQ is

(a) $\frac{x-2}{-1} = \frac{y-3}{2} = \frac{z-4}{5}$ (b) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$
(c) $\frac{x-2}{-1} = \frac{y-3}{-2} = \frac{z-4}{5}$ (d) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{5}$

28 If the points $(1, 2, 3)$ and $(2, -1, 0)$ lie on the opposite sides of the plane $2x + 3y - 2z = k$, then

(a) $k < 1$ (b) $k > 2$
(c) $k < 1$ or $k > 2$ (d) $1 < k < 2$

29 The equation of the plane containing the lines $2x - 5y + z = 3, x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$ is → **JEE Mains 2015**

(a) $2x + 6y + 12z = 13$ (b) $x + 3y + 6z = -7$
(c) $x + 3y + 6z = 7$ (d) $2x + 6y + 12z = -13$

30 The equation of a plane through the line of intersection of the planes $x + 2y = 3$, $y - 2z + 1 = 0$ and perpendicular to the first plane is → **JEE Mains 2013**

(a) $2x - y - 10z = 9$ (b) $2x - y + 7z = 11$
(c) $2x - y + 10z = 11$ (d) $2x - y - 9z = 10$

31 An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is → **AIEEE 2012**

(a) $x - 2y + 2z \pm 3 = 0$ (b) $x - 2y + 2z + 1 = 0$
(c) $x - 2y + 2z - 1 = 0$ (d) $x - 2y + 2z + 5 = 0$

32 Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b', c' from the origin, then → **AIEEE 2003**

(a) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$

(b) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(c) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(d) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

33 The distance of the point (1, – 5, 9) from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is
→ **JEE Mains 2016**

(a) $3\sqrt{10}$ (b) $10\sqrt{3}$ (c) $\frac{10}{\sqrt{3}}$ (d) $\frac{20}{3}$

34 Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is → **JEE Mains 2013**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

35 Find the planes bisecting the acute angle between the planes $x - y + 2z + 1 = 0$ and $2x + y + z + 2 = 0$.

(a) $x + z - 1 = 0$ (b) $x + z + 1 = 0$
(c) $x - z - 1 = 0$ (d) None of these

36 The angle between the lines $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane $x + y + 4 = 0$ is

(a) 0° (b) 30° (c) 45° (d) 90°

37 If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ is equal to → **AIEEE 2011**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$ (c) $\frac{5}{3}$ (d) $\frac{2}{3}$

38 The distance between the line $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ and the plane $\mathbf{r} \cdot (\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = 5$ is

(a) $\frac{10}{3\sqrt{3}}$ (b) $\frac{10}{9}$ (c) $\frac{10}{3}$ (d) $\frac{3}{10}$

39 Consider the following statements.

Statement I If the coordinates of the points A, B, C, D are $(1, 2, 3), (4, 5, 7), (-4, 3, -6)$ and $(2, 9, 2)$ respectively, then the angle between the lines AB and CD is $\frac{\pi}{6}$.

Statement II The straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-2}$ are parallel.

Choose the correct option.

(a) Statement I is true (b) Statement II is true
(c) Both statements are true (d) Both statements are false

40 Consider a line is perpendicular to the plane, then DR's of plane is proportional to the line.

Statement I The lines $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+1}{1}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{3}$ are coplanar and equation of the plane containing them is $5x + 2y - 3z - 8 = 0$

Statement II The line $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{3}$ is perpendicular to the plane $3x + 6y + 9z - 8 = 0$ and parallel to the plane $x + y - z = 0$

(a) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I
(b) Statement I is true; Statement II is true; Statement II is not a correct explanation of Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

41 **Statement I** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Statement II The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$. → **AIEEE 2011**

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

42 Consider the following statements

Statement I If the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$, then the equation of plane is $3x - 2y + 6z - 27 = 0$.

Statement II The equation of the plane through the points $(2, 1, 0), (3, -2, -2)$ and $(3, 1, 7)$ is $7x + 3y - z = 17$.

Choose the correct option.

(a) Statement I is true (b) Statement II is true
(c) Both statements are true (d) Both statements are false

43 **Statement I** The point $A(3, 1, 6)$ is the mirror image of the point $B(1, 3, 4)$ in the plane $x - y + z = 5$.

Statement II The plane $x - y + z = 5$ bisects the line segment joining $A(3, 1, 6)$ and $B(1, 3, 4)$. → **AIEEE 2010**

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

44 **Statement I** A point on the straight line $2x + 3y - 4z = 5$ and $3x - 2y + 4z = 7$ can be determined by taking $x = k$ and then solving the two equations for y and z, where k is any real number.

Statement II If $c' \neq kc$, then the straight line $ax + by + cz + d = 0$, $kax + kby + c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number.

(a) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I
(b) Statement I is true; Statement II is true; Statement II is not a correct explanation of Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

45 Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2},\ L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}.$$

Statement I The distance of the point $(1, 1, 1)$ from the plane passing through the point $(-1, -2, -1)$ and whose normal is perpendicular to both the lines L_1 and L_2 is $\frac{13}{5\sqrt{3}}$.

Statement II The unit vector perpendicular to both the lines L_1 and L_2 is $\frac{-\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}}{5\sqrt{3}}$.

(a) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I
(b) Statement I is true; Statement II is true; Statement II is not a correct explanation of Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The direction ratios of normal to the plane through $(1, 0, 0)$, $(0, 1, 0)$ which makes an angle $\frac{\pi}{4}$ with the plane $x + y = 3$ are

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$ (c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

2 The angle between the lines whose direction cosines are given by $2l - m + 2n = 0$, $lm + mn + nl = 0$, is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

3 If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is ➔ **JEE Mains 2018**

(a) $\frac{1}{4\sqrt{2}}$ (b) $\frac{1}{3\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

4 If the plane $x + y + z = 1$ is rotated through an angle 90° about its line of intersection with the plane $x - 2y + 3z = 0$, the new position of the plane is

(a) $x - 5y + 4z = 1$ (b) $x - 5y + 4z = -1$
(c) $x - 8y + 7z = 2$ (d) $x - 8y + 7z = -2$

5 A variable plane at a distance of 1 unit from the origin cut the coordinate axes at A, B and C. If the centroid $D(x, y, z)$ of ΔABC satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then k is equal to

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

6 The lines $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular, if ➔ **NCERT Exemplar**

(a) $pr + p'r' + 1 = 0$ (b) $pp' + rr' + 1 = 0$
(c) $pr + p'r' = 0$ (d) $pp' + rr' = 0$

7 A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection are given by

(a) $(3a, 3a, 3a)$, (a, a, a) (b) $(3a, 2a, 3a)$, (a, a, a)
(c) $(3a, 2a, 3a)$, $(a, a, 2a)$ (d) $(2a, 3a, 3a)$, $(2a, a, a)$

8 A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the coordinate planes. The length of a diagonal of the parallelopiped is

(a) 7 units (b) $\sqrt{38}$ units
(c) $\sqrt{155}$ units (d) None of these

9 The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$ having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is ➔ **JEE Mains 2017**

(a) $\frac{20}{\sqrt{74}}$ units (b) $\frac{10}{\sqrt{83}}$ units (c) $\frac{5}{\sqrt{83}}$ units (d) $\frac{10}{\sqrt{74}}$ units

10 Find the distance of the plane $x + 2y - z = 2$ from the point $(2, -1, 3)$ as measured in the direction with DR's $(2, 2, 1)$.

(a) 2 (b) -3 (c) -2 (d) 3

11 ΔABC is such that the mid-points of the sides BC, CA and AB are $(l, 0, 0)$, $(0, m, 0)$, $(0, 0, n)$, respectively. Then, $\frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2}$ is equal to

(a) 2 (b) 4 (c) 8 (d) 16

12 If α, β, γ and δ are the angles between a straight line with the diagonals of a cube, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ is equal to

(a) $\frac{5}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{4}$ (d) None of these

13 The equation of the line passing through the points $(3, 0, 1)$ and parallel to the planes $x + 2y = 0$ and $3y - z = 0$, is ➔ **NCERT Exemplar**

(a) $\frac{x-3}{-2} = \frac{y-0}{1} = \frac{z-1}{3}$ (b) $\frac{x-3}{1} = \frac{y-0}{-2} = \frac{z-1}{3}$
(c) $\frac{x-3}{3} = \frac{y-0}{1} = \frac{z-1}{-2}$ (d) None of these

14 The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is → **JEE Mains 2018**

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\sqrt{\frac{2}{3}}$

15 Let ABC be a triangle with vertices at points $A(2, 3, 5)$, $B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$ in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ) is equal to → **JEE Mains 2013**

(a) (10, 7) (b) (7, 5) (c) (7, 10) (d) (5, 7)

16 A plane passes through the point $(1, -2, 3)$ and is parallel to the plane $2x - 2y + z = 0$. The distance of the point $(-1, 2, 0)$ from the plane, is

(a) 2 (b) 3 (c) 4 (d) 5

17 The equation of the plane through the line intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$, which is perpendicular to plane $x - y + z = 0$, is

(a) $x + 2y + 3z - 4 = 0$
(b) $5x + 6y + 7z - 8 = 0$
(c) $120x + 144y + 168z - 5 = 0$
(d) $x - z + 2 = 0$

18 Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive X-axis, then $\cos\alpha$ is equal to

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

19 If the image of the point $P(1, -2, 3)$ in the plane $2x + 3y - 4z + 22 = 0$ measured parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to → **JEE Mains 2017**

(a) $3\sqrt{5}$ (b) $2\sqrt{42}$ (c) $\sqrt{42}$ (d) $6\sqrt{5}$

20 The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line → **JEE Mains 2014**

(a) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$
(b) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$
(c) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$.
(d) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

ANSWERS

SESSION 1

1. (a)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (c)
11. (a)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (d)
21. (b)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (c)	**30.** (c)
31. (a)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (d)	**40.** (b)
41. (b)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (a)					

SESSION 2

1. (b)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (d)
11. (c)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (d)	**19.** (b)	**20.** (a)

Hints and Explanations

SESSION 1

1 Since, S divides OG in the ratio $3 : -1$.

Then, $S = \left(\frac{9+3}{2}, \frac{-5+9}{2}, \frac{-3-1}{2}\right)$

$= (6, 2, -2)$

2 A line makes angle θ with X-axis and Z-axis and β with Y-axis.

$\therefore \quad l = \cos\theta,\ m = \cos\beta,\ n = \cos\theta$

$\because \quad l^2 + m^2 + n^2 = 1$

$\therefore \quad \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow \quad 2\cos^2\theta = 1 - \cos^2\beta$

$\Rightarrow \quad 2\cos^2\theta = \sin^2\beta \qquad \text{...(i)}$

But it is given that,

$\sin^2\beta = 3\sin^2\theta \qquad \text{...(ii)}$

From Eqs. (i) and (ii), we get

$3\sin^2\theta = 2\cos^2\theta$

$\Rightarrow \quad 3(1 - \cos^2\theta) = 2\cos^2\theta$

$\Rightarrow \quad 3 = 5\cos^2\theta$

$\therefore \quad \cos^2\theta = \frac{3}{5}$

3 We know that,

$\cos^2\theta + \cos^2\theta + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\gamma = -\cos 2\theta$

$\Rightarrow \quad \cos 2\theta \le 0$

$\therefore \quad \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

4 Projection of a vector on coordinate axes are

$x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$

$\Rightarrow \quad x_2 - x_1 = 6,\ y_2 - y_1 = -3,\ z_2 - z_1 = 2$

Now, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{36 + 9 + 4} = 7$

So, the DC's of the vector are $\frac{6}{7}, -\frac{3}{7}$ and $\frac{2}{7}$.

5 Given that, the projections of a line segment on the X, Y and Z-axes in $3D$-space are, $lr = 2$, $mr = 3$ and $nr = 6$

$\because (lr)^2 + (mr)^2 + (nr)^2 = (2)^2 + (3)^2 + (6)^2$

$\Rightarrow \quad (l^2 + m^2 + n^2)r^2 = 4 + 9 + 36$

$\Rightarrow \quad r^2 = 49 \Rightarrow r = 7$

6 Let $\mathbf{r}$ be inclined at an angle α to each axis, then $l = m = n = \cos\alpha$

Since, $\quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 3\cos^2\alpha = 1$

If α is acute, then $l = m = n = \frac{1}{\sqrt{3}}$ and $|\mathbf{r}| = 6$

$\therefore \quad \mathbf{r} = |\mathbf{r}|\,(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$

$= 6\left(\frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}\right)$

$= 2\sqrt{3}\,(\mathbf{i} + \mathbf{j} + \mathbf{k})$

If α is obtuse, then

$l = m = n = -\frac{1}{\sqrt{3}}$ and $|\mathbf{r}| = 6$

$\therefore \quad \mathbf{r} = |\mathbf{r}|\,(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$

$= 6\left(-\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}\right)$

$= -2\sqrt{3}\,(\mathbf{i} + \mathbf{j} + \mathbf{k})$

7 Equation of L_1 is

$\mathbf{r} = 3\mathbf{i} + \lambda(-\mathbf{i} + \mathbf{j} + \mathbf{k})$

$= (3 - \lambda)\mathbf{i} + \lambda\mathbf{j} + \lambda\mathbf{k}$

Equation of L_2 is

$\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \mu(\mathbf{i} + \mathbf{k})$

$= (1 + \mu)\mathbf{i} + \mathbf{j} + \mu\mathbf{k}$

For point of intersection, we get

$\lambda = \mu = 1 \Rightarrow \mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$

8 Equation of line passing through $(5, 1, a)$ and $(3, b, 1)$ is

$\frac{x-3}{5-3} = \frac{y-b}{1-b} = \frac{z-1}{a-1} \qquad \text{...(i)}$

$\left[\because \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}\right]$

Point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ satisfies Eq. (i), we get

$-\frac{3}{2} = \frac{\frac{17}{2} - b}{1 - b} = \frac{-\frac{13}{2} - 1}{a - 1}$

$\Rightarrow \quad a - 1 = \frac{\left(-\frac{15}{2}\right)}{\left(-\frac{3}{2}\right)} = 5 \Rightarrow a = 6$

Also, $-3(1 - b) = 2\left(\frac{17}{2} - b\right)$

$\Rightarrow 3b - 3 = 17 - 2b \Rightarrow 5b = 20 \Rightarrow b = 4$

9 The given equations of lines can be rewritten as

$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$\therefore$ Angle between the lines is

$\theta = \cos^{-1}\left(\frac{3 \times 2 + 2(-12) - 6(-3)}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{(2)^2 + (-12)^2 + (-3)^2}}\right)$

$\left[\because \cos\theta = \frac{a_1 \cdot a_2 + b_2 \cdot b_2 + c_1 \cdot c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right]$

$= \cos^{-1}(0) = 90°$

10 If three edges of the cube are along x, y and z, then diagonal has DR's 1, 1, 1 and edge along X-axis has DR's 1, 0, 0. The angle between them is

$\cos^{-1}\frac{1}{\sqrt{3}} = \tan^{-1}\sqrt{2}$

11 We know that, angle between two lines is

$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$l + m + n = 0$

$\Rightarrow \quad l = -(m + n)$

$\Rightarrow \quad (m + n)^2 = l^2$

$\Rightarrow \quad m^2 + n^2 + 2mn = m^2 + n^2$

$[\because l^2 = m^2 + n^2, \text{ given}]$

$\Rightarrow \quad 2mn = 0$

When, $m = 0 \Rightarrow l = -n$

Hence, (l, m, n) is $(1, 0, -1)$.

When $n = 0$, then $l = -m$

Hence, (l, m, n) is $(1, 0, -1)$.

$\therefore \cos\theta = \frac{1 + 0 + 0}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

12 Since, the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane $lx + my - z = 9$, therefore we have $2l - m - 3 = 0$

[$\because$ normal will be perpendicular to the line]

$\Rightarrow \quad 2l - m = 3 \qquad \text{...(i)}$

and $\quad 3l - 2m + 4 = 9$

[$\because$ point $(3, -2, -4)$ lies on the plane]

$\Rightarrow \quad 3l - 2m = 5 \qquad \text{...(ii)}$

On solving Eqs. (i) and (ii), we get

$l = 1$ and $m = -1$

$\therefore \quad l^2 + m^2 = 2$

13 Let the direction cosine of the line perpendicular to two given lines is (l, m, n), then $l + 2m + 3n = 0$ and $-2l + \frac{m}{2} + \frac{n}{3} = 0$

From the above equation,

$\frac{l}{2 \times \frac{1}{3} - \frac{1}{2} \times 3} = -\frac{m}{-3 \times (-2) - 1 \times \frac{1}{3}}$

$= \frac{n}{1 \times \frac{1}{2} - 2(-2)}$

$\Rightarrow \frac{l^2}{\frac{25}{36}} = \frac{m^2}{\frac{361}{9}} = \frac{n^2}{\frac{81}{4}} = \frac{1}{\frac{25}{36} + \frac{361}{9} + \frac{81}{4}}$

$\therefore \quad l = \sqrt{\frac{25}{2198}},\ m = \frac{19}{\sqrt{2198}},\ n = \sqrt{\frac{729}{2198}}$

14 Let L be foot of perpendicular from $P(0, 2, 3)$ on the line

$$\frac{x-(-3)}{5} = \frac{y-1}{2} = \frac{z-(-4)}{3} = t \quad \ldots(\text{i})$$

Any point on Eq. (i) is

$L(-3+5t, 1+2t, -4+3t)$.

Then, DR's of PL are

$(-3+5t-0, 1+2t-2, -4+3t-3)$

or $(5t-3, 2t-1, 3t-7)$.

Since, PL is perpendicular to Eq. (i), therefore $5(5t-3)+2(2t-1)$

$+3(3t-7) = 0 \Rightarrow t = 1$

So, the coordinate of L is $(2, 3, -1)$.

15 The vector joining the points is $\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. Its projection along the vector $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$$= \frac{|(\mathbf{i}-3\mathbf{j}+\mathbf{k})\cdot(2\mathbf{i}+\mathbf{j}-2\mathbf{k})|}{\sqrt{2^2+1^2+2^2}}$$

$$= \frac{|2-3-2|}{3} = 1$$

16 The given lines are

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

and $\dfrac{x-3}{1} = \dfrac{y-5}{-2} = \dfrac{z-7}{1}$

For line Ist DR's $= (7, -6, 1)$ and it passes through $(-1, -1, -1)$, then equation of given lines (in vector form) is

$\mathbf{r}_1 = -\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(7\mathbf{i} - 6\mathbf{j} + \mathbf{k})$

Similarly, $\mathbf{r}_2 = 3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

which are of the form $\mathbf{r}_1 = \mathbf{a}_1 + \lambda\mathbf{b}_1$ and $\mathbf{r}_2 = \mathbf{a}_2 + \mu\mathbf{b}_2$ where,

$\mathbf{a}_1 = -\mathbf{i} - \mathbf{j} - \mathbf{k}, \mathbf{b}_1 = 7\mathbf{i} - 6\mathbf{j} + \mathbf{k}$

and $\mathbf{a}_2 = 3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}, \mathbf{b}_2 = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Now, $\mathbf{a}_2 - \mathbf{a}_1 = (3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k})$

$-(-\mathbf{i} - \mathbf{j} - \mathbf{k}) = 4\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$

and $\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$= \mathbf{i}(-6+2) - \mathbf{j}(7-1) + \mathbf{k}(-14+6)$

$= -4\mathbf{i} - 6\mathbf{j} - 8\mathbf{k}$

$|\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$

$= \sqrt{16+36+64}$

$= \sqrt{116} = 2\sqrt{29}$

So, the shortest distance between the given lines

$$d = \left|\frac{(\mathbf{b}_1 \times \mathbf{b}_2)\cdot(\mathbf{a}_2 - \mathbf{a}_1)}{|\mathbf{b}_1 \times \mathbf{b}_2|}\right|$$

$$= \frac{|(-4\mathbf{i} - 6\mathbf{j} - 8\mathbf{k})\cdot(4\mathbf{i} + 6\mathbf{j} + 8\mathbf{k})|}{2\sqrt{29}}$$

$$= \frac{|(-4)\times 4 + (-6)\times 6 + (-8)\times 8|}{2\sqrt{29}}$$

$$= \frac{|-16-36-64|}{2\sqrt{29}} = \frac{116}{2\sqrt{29}}$$

$$= \frac{58}{\sqrt{29}} = 2\sqrt{29} \text{ units}$$

17 Let one vertex of the parallelopiped be at the origin O and three coterminous edges OA, OB and OC be along OX, OY and OZ, respectively. The coordinates of the vertices of the parallelopiped are marked in figure.

The edges which do not meet the diagonal OF are AH, AD and BD and their parallels are BE, CE and CH, respectively.

The vector equation of the diagonal OF is

$$\mathbf{r} = 0 + \lambda(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \quad \ldots(\text{i})$$

The vector equation of the edge BD is

$$\mathbf{r} = b\,\mathbf{j} + \mu\, a\mathbf{i} \quad \ldots(\text{ii})$$

We have,

$(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times a\mathbf{i} = ba(\mathbf{j} \times \mathbf{i})$

$+ ca(\mathbf{k} \times \mathbf{i})$

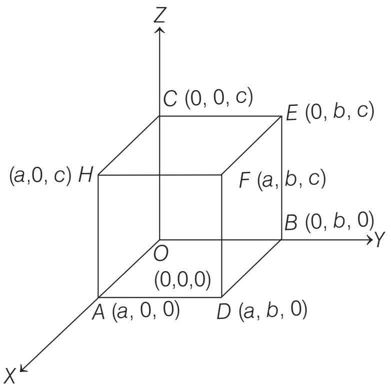
$= -ba\,\mathbf{k} + ca\,\mathbf{j}$

$\therefore |(a\mathbf{i} \times b\mathbf{j} \times c\mathbf{k}) \times a\mathbf{i}| = \sqrt{b^2a^2 + c^2a^2}$

and $\{(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times a\mathbf{i}\}\cdot(b\mathbf{j} - \mathbf{0})$

$= (-ba\,\mathbf{k} + ca\,\mathbf{j})\cdot b\mathbf{j}$

$= abc$



Thus, the shortest distance between Eqs. (i) and (ii) is given by

$$SD = \frac{|\{(a\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}) \times a\mathbf{i}\}\cdot(b\mathbf{j} - \mathbf{0})|}{|(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times a\mathbf{i}|}$$

$$= \frac{abc}{\sqrt{b^2a^2 + c^2a^2}} = \frac{bc}{\sqrt{b^2 + c^2}}$$

Similarly, it can be shown that the shortest distance between OF and AD is $\dfrac{ca}{\sqrt{a^2 + c^2}}$ and that between OF and AH is $\dfrac{ab}{\sqrt{a^2 + b^2}}$.

18 Let $L_1 : \dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4} = p$

and $L_2 : \dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z-0}{1} = q$

$\Rightarrow$ Any point P on line L_1 is of type $P(2p+1, 3p-1, 4p+1)$ and any point Q on line L_2 is of type $Q(q+3, 2q+k, q)$

Since, L_1 and L_2 are intersecting each other, hence both point P and Q should coincide at the point of intersection, i.e. corresponding coordinates of P and Q should be same.

$2p+1 = q+3, 4p+1 = q$

and $3p-1 = 2q+k$

On solving $2p+1 = q+3$ and $4p+1 = q$, we get the values of p and q as

$$p = \frac{-3}{2} \text{ and } q = -5$$

On substituting the values of p and q in the third equation $3p-1 = 2q+k$, we get

$$\therefore 3\left(\frac{-3}{2}\right) - 1 = 2(-5) + k \Rightarrow k = \frac{9}{2}$$

19 The given straight line can be rewritten as

$$\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s$$

and $\dfrac{x-0}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{-2} = t$

These two lines are coplanar, if

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1-0 & -3-1 & 1-2 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$\Rightarrow 1(2\lambda - 2\lambda) + 4(-2-\lambda) - 1(2+\lambda) = 0$

$\Rightarrow -8 - 4\lambda - 2 - \lambda = 0$

$\Rightarrow -10 = 5\lambda \Rightarrow \lambda = -2$

20 Given equation of line is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda \quad [\text{say}] \ldots(\text{i})$$

and equation of plane is

$$x - y + z = 16 \quad \ldots(\text{ii})$$

Any point on the line (i) is

$(3\lambda+2, 4\lambda-1, 12\lambda+2)$

Let this point be point of intersection of the line and plane.

$\therefore (3\lambda+2) - (4\lambda-1) + (12\lambda+2) = 16$

$\Rightarrow 11\lambda + 5 = 16$

$\Rightarrow 11\lambda = 11 \Rightarrow \lambda = 1$

$\therefore$ Point of intersection is $(5, 3, 14)$.

Now, distance between the points $(1, 0, 2)$ and $(5, 3, 14)$

$= \sqrt{(5-1)^2 + (3-0)^2 + (14-2)^2}$

$= \sqrt{16+9+144} = \sqrt{169} = 13$

21 $\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2 45^\circ + \cos^2 60^\circ + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \dfrac{1}{2} - \dfrac{1}{4} = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

$\Rightarrow \cos\gamma = \dfrac{1}{2}$

$\therefore$ Direction Ratio's of normal to the plane is $< \cos 45°; \cos 60°, \frac{1}{2} >$

$$= < \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2} >$$

$\therefore$ Equation of plane passing through $(\sqrt{2}, -1, 1)$ is

$$\frac{1}{\sqrt{2}}(x - \sqrt{2}) + \frac{1}{2}(y + 1) + \frac{1}{2}(z - 1) = 0$$

$$\Rightarrow 2(x - \sqrt{2}) + \sqrt{2}(y + 1) + \sqrt{2}(z - 1) = 0$$

$$\Rightarrow \sqrt{2}(x - \sqrt{2}) + (y + 1) + (z - 1) = 0$$

$$\Rightarrow \sqrt{2}x - 2 + y + 1 + z - 1 = 0$$

$$\Rightarrow \sqrt{2}x + y + z = 2$$

22 Let foot of perpendicular $Q(x, y, z)$ from $O(0, 0, 0)$

$$\frac{x - 0}{4} = \frac{y - 0}{-3} = \frac{z - 0}{1} = -\frac{\{4(0) - 3(0) + 1(0) + 13\}}{4^2 + 3^2 + 1^2}$$

$$\Rightarrow \frac{x}{4} = \frac{y}{-3} = \frac{z}{1} = \frac{-13}{26} = -\frac{1}{2}$$

$$x = -2, \ y = \frac{3}{2}, \ z = -\frac{1}{2}$$

$$\therefore Q\left(-2, \frac{3}{2}, -\frac{1}{2}\right)$$

$$\therefore PQ = \sqrt{(-1 + 2)^2 + \left(1 - \frac{3}{2}\right)^2 + \left(-6 + \frac{1}{2}\right)^2}$$

$$= \sqrt{1 + \frac{1}{4} + \frac{121}{4}} = \frac{\sqrt{126}}{2} = 3\sqrt{\frac{7}{2}}$$

23 Required equation of plane is

$$\begin{vmatrix} x - 1 & y - 1 & z - 1 \\ -3 & -3 & 1 \\ 0 & -2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x - 3y - 6z + 8 = 0$$

Since, the intercepts are $8, \frac{8}{3}, \frac{8}{6}$.

So, their sum is 12.

24 Equation of plane through three points $(2, 2, 1)$, $(3, 0, 1)$ and $(4, -1, 0)$ is

$[(\mathbf{r} - \mathbf{i} + 2\mathbf{j} + \mathbf{k})] \cdot [(\mathbf{i} - 2\mathbf{j}) \times (\mathbf{i} - \mathbf{j} - \mathbf{k})] = 0$

i.e. $\quad \mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} + \mathbf{k}) = 7$

or $\quad 2x + y + z - 7 = 0 \quad \ldots$(i)

Equation of line through $(3, -4, -5)$ and $(2, -3, 1)$ is

$$\frac{x - 3}{-1} = \frac{y + 4}{1} = \frac{z + 5}{6} \quad \ldots\text{(ii)}$$

Any point on line (ii) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$. This point lies on plane (i).

Therefore,

$$2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0$$

$$\Rightarrow \lambda = 2$$

Hence, the required point is $(1, -2, 7)$.

25 Since, the vertices of the tetrahedron are $(0, 0, 0), (3, 0, 0), (0, 2, 0)$ and $(0, 0, 6)$.

$\therefore$ Volume of tetrahedron

$$= \frac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{vmatrix} = 6$$

26 The equation of the plane parallel to the plane $3x - 4y + 5z = 4$ is

$3x - 4y + 5z + k = 0$

Since, this plane passes through $(2, 1, 5)$. On substituting coordinates $(2, 1, 5)$, we get

$3 \times 2 - 4 \times 1 + 5 \times 5 + k = 0 \Rightarrow k = -27$

So, the equation of plane is

$3x - 4y + 5z - 27 = 0$.

27 Since Q is the image of P, therefore PQ is perpendicular to the plane $x - 2y + 5z = 6$.

$\therefore$ Required equation of line is

$$\frac{x - 2}{1} = \frac{y - 3}{-2} = \frac{z - 4}{5}$$

28 On substituting the coordinates of the points in the equation $2x + 3y - 2z - k = 0$, we get

$$(2 + 6 - 6 - k)(4 - 3 - k) < 0$$

$$\Rightarrow (k - 1)(k - 2) < 0$$

$$\therefore 1 < k < 2$$

29 Let equation of plane containing the lines $2x - 5y + z = 3$ and $x + y + 4z = 5$ be $(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

$$\Rightarrow (2 + \lambda)x + (\lambda - 5)y + (4\lambda + 1)z - 3 - 5\lambda = 0 \quad \ldots\text{(i)}$$

This plane is parallel to the plane $x + 3y + 6z = 1$.

$$\therefore \frac{2 + \lambda}{1} = \frac{\lambda - 5}{3} = \frac{4\lambda + 1}{6}$$

On taking first two equalities, we get

$$6 + 3\lambda = \lambda - 5 \Rightarrow 2\lambda = -11$$

$$\Rightarrow \lambda = -\frac{11}{2}$$

On taking last two equalities, we get

$$6\lambda - 30 = 3 + 12\lambda$$

$$\Rightarrow -6\lambda = 33 \Rightarrow \lambda = -\frac{11}{2}$$

So, the equation of required plane is

$$\left(2 - \frac{11}{2}\right)x + \left(\frac{-11}{2} - 5\right)y + \left(-\frac{44}{2} + 1\right)z - 3 + 5 \times \frac{11}{2} = 0$$

$$\Rightarrow -\frac{7}{2}x - \frac{21}{2}y - \frac{42}{2}z + \frac{49}{2} = 0$$

$$\Rightarrow x + 3y + 6z - 7 = 0$$

30 Intersection of two planes is

$$(x + 2y - 3) + \lambda(y - 2z + 1)$$

$$\Rightarrow x + (2 + \lambda)y - 2\lambda z + \lambda - 3 = 0$$

$$\therefore 1(1) + 2(2 + \lambda) + 0(-2\lambda) = 0 \Rightarrow \lambda = -\frac{5}{2}$$

$$\therefore (x + 2y - 3) - \frac{5}{2}(y - 2z + 1) = 0$$

$$\Rightarrow 2x + 4y - 6 - 5y + 10z - 5 = 0$$

$$\Rightarrow 2x - y + 10z - 11 = 0$$

$$\Rightarrow 2x - y + 10z = 11$$

31 Given, a plane $P : x - 2y + 2z - 5 = 0$

Equation of family of planes parallel to the given plane P is

$$Q : x - 2y + 2z + d = 0$$

Also, perpendicular distance of Q from origin is 1 unit.

$$\Rightarrow \left|\frac{0 - 2(0) + 2(0) + d}{\sqrt{1^2 + 2^2 + 2^2}}\right| = 1$$

$$\Rightarrow \left|\frac{d}{3}\right| = 1 \Rightarrow d = \pm 3$$

Hence, the required equation of the plane parallel to P and at unit distance from origin is $x - 2y + 2z \pm 3 = 0$.

32 Consider OX, OY, OZ and Ox, Oy, Oz are two systems of rectangular axes.

Let their corresponding equations of plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \ldots\text{(i)}$$

and $$\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1 \quad \ldots\text{(ii)}$$

Length of perpendicular from origin to Eqs. (i) and (ii) must be same.

$$\therefore \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$$

33 Equation of PQ is

$$\frac{x - 1}{1} = \frac{y + 5}{1} = \frac{z - 9}{1} = \lambda$$

So, $x = \lambda + 1$, $y = \lambda - 5$ and $z = \lambda + 9$ lies on the plane $x - y + z = 5$.

$\Rightarrow \lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\therefore \lambda = -10$

So, the coordinate of Q is $(-9, -15, -1)$ and coordinate of P is $(1, -5, 9)$.

$\therefore |PQ| = \sqrt{(10)^2 + (10)^2 + (10)^2} = 10\sqrt{3}$

34 Given planes are,

$$2x + y + 2z - 8 = 0$$

and $$2x + y + 2z + \frac{5}{2} = 0$$

Distance between two planes

$$= \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2 + c^2}} = \left|\frac{-8 - \frac{5}{2}}{\sqrt{2^2 + 1^2 + 2^2}}\right|$$

$$= \frac{\frac{21}{2}}{3} = \frac{7}{2}$$

35 Now, $a_1 a_2 + b_1 b_2 + c_1 c_2 = 2 - 1 + 2 > 0$.

The acute angle bisecting plane is

$$x - y + 2z + 1 = -(2x + y + z + 2)$$

i.e. $x + z + 1 = 0$

36 DR's of line are 2, 1, −2 and DR's of normal to the plane are 1, 1, 0.

$\therefore$ Therefore, their DC's are $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$ and $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ respectively.

Now, let θ be the angle b/w line and the plane, then

$$\cos(90° - \theta) = \frac{2}{3}\cdot\frac{1}{\sqrt{2}} + \frac{1}{3}\cdot\frac{1}{\sqrt{2}} + \left(-\frac{2}{3}\right)\cdot 0$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

37 Angle between straight line $\mathbf{r} = \mathbf{a} + \lambda\,\mathbf{b}$ and plane $\mathbf{r}\cdot\mathbf{n} = \mathbf{d}$,

$$\sin\theta = \frac{\mathbf{b}\cdot\mathbf{n}}{|\mathbf{b}||\mathbf{n}|}$$

$$\therefore \quad \sin\theta = \frac{(\mathbf{i} + 2\,\mathbf{j} + \lambda\,\mathbf{k})\cdot(\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k})}{\sqrt{1 + 4 + \lambda^2}\,\sqrt{1 + 4 + 9}}$$

$$\Rightarrow \sin\theta = \frac{5 + 3\lambda}{\sqrt{\lambda^2 + 5}\cdot\sqrt{14}}$$

Given, $\cos\theta = \sqrt{\frac{5}{14}}$

$$\therefore \quad \sin\theta = \frac{3}{\sqrt{14}}$$

$$\Rightarrow \quad \frac{3}{\sqrt{14}} = \frac{5 + 3\lambda}{\sqrt{\lambda^2 + 5}\cdot\sqrt{14}}$$

$\Rightarrow$ $9(\lambda^2 + 5) = 9\lambda^2 + 30\lambda + 25$

$\Rightarrow$ $9\lambda^2 + 45 = 9\lambda^2 + 30\lambda + 25$

$\Rightarrow$ $30\lambda = 20$

$\therefore$ $\lambda = \frac{2}{3}$

38 Clearly, given line is parallel to the plane.

Given point on the line is $A(2, -2, 3)$ and a point on the plane is $B(0, 0, 5)$

$\therefore$ $\mathbf{AB} = (2 - 0)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 5)\mathbf{k}$

$= 2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$

Now, required distance = Projection of $\mathbf{AB}$ or $\mathbf{i} + 5\mathbf{j} + \mathbf{k}$

$$= \left|\frac{(2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})\cdot(\mathbf{i} + 5\mathbf{j} + \mathbf{k})}{\sqrt{1 + 25 + 1}}\right|$$

$$= \left|\frac{2 - 10 - 2}{\sqrt{27}}\right|$$

$$= \frac{10}{3\sqrt{3}}$$

39 I. $\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$

$= (4\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}) - (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$

$= 3\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

$\mathbf{CD} = \mathbf{OD} - \mathbf{OC}$

$= (2\mathbf{i} + 9\mathbf{j} + 2\mathbf{k}) - (-4\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$

$= 6\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$

$$\because \cos\theta = \left|\frac{\mathbf{AB}\cdot\mathbf{CD}}{|\mathbf{AB}||\mathbf{CD}|}\right|$$

$$\cos\theta = \left|\frac{(3\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})\cdot(6\mathbf{i} + 6\mathbf{j} + 8\mathbf{k})}{\sqrt{3^2 + 3^2 + 4^2}\,\sqrt{6^2 + 6^2 + 8^2}}\right|$$

$$= \left|\frac{18 + 18 + 32}{\sqrt{34}\sqrt{136}}\right| = \left|\frac{68}{2\times 34}\right| = 1$$

$\therefore$ $\theta = \cos^{-1} 1 = 0$

Hence, Statement I is false.

II. Given, $a_1 = 1, b_1 = 2, c_1 = 3$

$a_2 = 2, b_2 = 2, c_2 = -2$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{2}$$

$$\because \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, given lines are not parallel.

Therefore, Statement II is false.

40 **Statement I** The equation of the plane containing them is

$$\begin{vmatrix} x - 1 & y & z + 1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ $-(5x + 2y - 3z - 8) = 0$

Statement II Here, $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$

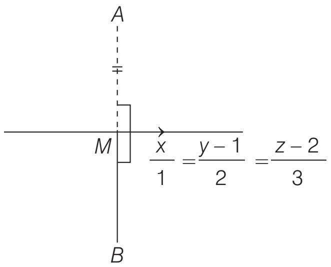
$$\Rightarrow \frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

and $1(1) + 2(1) + 3(-1) = 0$

$\therefore$ Statement II is true.

41 Since, mid-point on AB is M (1, 3, 5).

which lies on $\frac{x}{1} = \frac{y - 1}{2} = \frac{z - 2}{3}$.



$$\therefore \quad \frac{1}{1} = \frac{3 - 1}{2} = \frac{5 - 2}{3}$$

$\Rightarrow$ $1 = 1 = 1$

Hence, Statement II is true.

Also, direction ratio of AB is

$(1 - 1, 6 - 0, 3 - 7) = (0, 6, -4)$ ...(i)

and direction ratio of straight line is

$(1, 2, 3)$ ... (ii)

These two lines are perpendicular, if

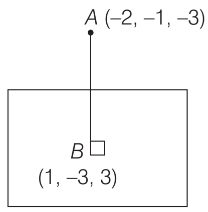
$0(1) + 6(2) - 4(3) = 12 - 12 = 0$

Hence, Statement I is true.

42 I. $\mathbf{N} = \mathbf{OB} - \mathbf{OA}$

$= (\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}) - (-2\mathbf{i} - \mathbf{j} - 3\mathbf{k})$

$= 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$



Equation of a plane is given by

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

$\Rightarrow 3(x - 1) + (-2)(y + 3) + 6(z - 3) = 0$

$\Rightarrow$ $3x - 3 - 2y - 6 + 6z - 18 = 0$

$\Rightarrow$ $3x - 2y + 6z - 27 = 0$

II. Equation of any plane through $(2, 1, 0)$ is

$a(x - 2) + b(y - 1) + c(z - 0) = 0$ ...(i)

Since, it passes through the points (3,-2,-2) and (3,1,7). Then, we get

$a - 3b - 2c = 0$ ...(ii)

and $a + 0b + 7c = 0$ ...(iii)

On solving Eqs.(ii) and (iii) by cross-multiplication, we get

$a = 7\lambda, b = 3\lambda, c = -\lambda$

On substituting the value of a, b, c in Eq. (i), we get

$7\lambda(x - 2) + 3\lambda(y - 1) - \lambda z = 0$

$\Rightarrow$ $7x + 3y - z = 17$

This is the required equation of the plane.

43 The image of the point (3, 1, 6) with respect to the plane $x - y + z = 5$ is

$$\frac{x - 3}{1} = \frac{y - 1}{-1} = \frac{z - 6}{1}$$

$$= \frac{-2(3 - 1 + 6 - 5)}{1 + 1 + 1}$$

$$\Rightarrow \frac{x - 3}{1} = \frac{y - 1}{-1} = \frac{z - 6}{1} = -2$$

$\Rightarrow$ $x = 3 - 2 = 1$

$y = 1 + 2 = 3$

$z = 6 - 2 = 4$

which shows that Statement I is true.

We observe that the line segment joining the points $A(3, 1, 6)$ and $B(1, 3, 4)$ has direction ratios 2, − 2, 2 which are proportional to 1, − 1, 1 the direction ratios of the normal to the plane. Hence, Statement II is true.

Thus, the Statements I and II are true and Statement II is correct explanation of Statement I.

44 **Statement I** $3y - 4z = 5 - 2k$

$-2y + 4z = 7 - 3k$

$\therefore$ $x = k, y = 12 - 5k$ and

$z = \frac{31 - 13k}{4}$ is a point on the line for all real values of k.

Statement I is true.

Statement II Direction ratios of the straight line are

$< bc' - kbc, kac - ac', 0 >$ and direction ratios of normal the plane are $< 0, 0, 1 >$.

Now, $0 \times (bc' - kbc) + 0 \times (kac - ac') + 1 \times 0 = 0$

Hence, the straight line is parallel to the plane.

45 **Statement II** Lines L_1 and L_2 are parallel to the vectors $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, respectively. The unit vector perpendicular to both L_1 and L_2 is

$$\frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|} = \frac{-\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}}{\sqrt{1 + 49 + 25}} = \frac{-\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}}{5\sqrt{3}}$$

Hence, Statement II is true.

Statement I Plane is

$-(x + 1) - 7(y + 2) + 5(z + 1) = 0$, whose distance from $(1, 1, 1)$ is $\frac{13}{5\sqrt{3}}$.

Hence, Statement I is true.

Thus, statement I is true, statement II is true; Statement II is a correct explanation of Statement I.

SESSION 2

1 Let the equation of plane in intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Since, it passes through the point $(1, 0, 0)$ and $(0, 1, 0)$.

$\therefore$ Equation of plane is $\frac{x}{1} + \frac{y}{1} + \frac{z}{c} = 1$

DR's of normal are $1, 1, \frac{1}{c}$ and of given plane are 1,1,0.

Now, $\cos\frac{\pi}{4} = \frac{1 \cdot 1 + 1 \cdot 1 + \frac{1}{c} \cdot 0}{\left(\sqrt{\frac{1}{c^2} + 2}\right)\sqrt{2}}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{2}{\left(\sqrt{\frac{1}{c^2} + 2}\right)\sqrt{2}}$

$\Rightarrow \quad \frac{1}{c^2} + 2 = 4 \quad \Rightarrow \quad c^2 = \frac{1}{2}$

$\therefore \quad c = \frac{1}{\sqrt{2}}$

So, the DR's of normal are $1, 1, \sqrt{2}$.

2 On eliminating m from given equations, we get

$2(l + n)^2 + nl = 0$ [$\because$ put $m = 2l + 2n$]

$\Rightarrow \quad (2l + n)(l + 2n) = 0$

$\Rightarrow \quad n = -2l \Rightarrow m = -2l$

or $\quad l = -2n$

$\Rightarrow \quad m = -2n$

The DR's are $1, -2, -2$ and $-2, -2, 1$.

Now, $1(-2) - 2(-2) - 2(1) = 0$

Hence, lines are perpendicular.

So, angle between them is $\pi/2$.

3 L_1 is the line of intersection of the plane $2x - 2y + 3z - 2 = 0$ and $x - y + z + 1 = 0$ and L_2 is the line of intersection of the plane $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$

Since L_1 is parallel to

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} + \hat{j}$$

L_2 is parallel to

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}$$

Also, L_2 passes through $\left(\frac{5}{7}, \frac{8}{7}, 0\right)$.

[put $z = 0$ in last two planes]

So, equation of plane is

$$\begin{vmatrix} x - \frac{5}{7} & y - \frac{8}{7} & z \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$$

$\Rightarrow \quad 7x - 7y + 8z + 3 = 0$

Now, perpendicular distance from origin is

$$\left|\frac{3}{\sqrt{7^2 + 7^2 + 8^2}}\right| = \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$$

4 The new position of plane is

$x - 2y + 3z + \lambda(x + y + z - 1) = 0$

$\Rightarrow (1 + \lambda)x + (\lambda - 2)y + (\lambda + 3)z - \lambda = 0$

Since, it is perpendicular to $x + y + z - 1 = 0$.

$\therefore 1 + \lambda + \lambda - 2 + \lambda + 3 = 0$

$\Rightarrow \lambda = -\frac{2}{3}$

Hence, required plane is $x - 8y + 7z = -2$.

5 Let plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ cuts the axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$.

Centroid of plane ABC is $D\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$.

Distance of the plane from the origin

$d = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = 1$ [given]

$\Rightarrow \quad 1 = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$

$\therefore \quad D(x, y, z)$

$\Rightarrow \quad a = 3x, b = 3y, c = 3z$

$\Rightarrow \quad \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 9$

Hence, the value of k is 9.

6 Given lines are

$x = py + q, z = ry + s$

$\Rightarrow \quad \frac{x - q}{p} = y = \frac{z - s}{r}$

and $\quad x = p'y + q', z = r'y + s'$

$\Rightarrow \quad \frac{x - q'}{p'} = y = \frac{z - s'}{r'}$

Two lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$

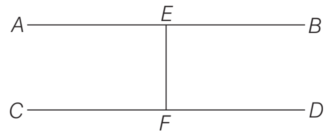
and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ are perpendicular, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\therefore$ Given lines are perpendicular, if

$pp' + rr' = 0$

7 Let the equation of line AB be

$\frac{x - 0}{1} = \frac{y + a}{1} = \frac{z - 0}{1} = k$ [say]



Any point on the line is $F(k, k - a, k)$.

Also, the equation of other line CD is

$\frac{x + a}{2} = \frac{y - 0}{1} = \frac{z - 0}{1} = \lambda$ [say]

Any point on the line is $E(2\lambda - a, \lambda, \lambda)$.

Direction ratios of EF are $[(k - 2\lambda + a), (k - a - \lambda), (k - \lambda)]$.

Since, it is given that direction ratios of EF are proportional to 2, 1, 2.

$\therefore \quad \frac{k - 2\lambda + a}{2} = \frac{k - \lambda - a}{1} = \frac{k - \lambda}{2}$

On solving first and second fractions, we get

$k - 2\lambda + a = 2k - 2\lambda - 2a$

$\Rightarrow \quad k = 3a \quad \ldots(i)$

On solving second and third fractions, we get

$2k - 2\lambda - 2a = k - \lambda$

$\Rightarrow \quad k - \lambda = 2a$

$\Rightarrow \quad \lambda = 3a - 2a$ [from Eq. (i)]

$\therefore \lambda = a$

Hence, coordinates of E are $(3a, 2a, 3a)$ and coordinates of F are (a, a, a).

8 A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the coordinate planes.

Let a, b and c be the lengths of edges, then

$a = 5 - 2 = 3, b = 9 - 3 = 6$

and $\quad c = 7 - 5 = 2$

So, the length of diagonal of a parallelopiped

$$= \sqrt{a^2 + b^2 + c^2}$$
$$= \sqrt{9 + 36 + 4}$$
$$= \sqrt{49} = 7 \text{ units}$$

9 Given, equations of lines are

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$$

and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$

Let $\mathbf{n}_1 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\mathbf{n}_2 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore$ Any vector $\mathbf{n}$ perpendicular to both $\mathbf{n}_1, \mathbf{n}_2$ is given by

$$\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$$

$$\Rightarrow \mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$$

$$= 5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$\therefore$ Equation of a plane passing through $(1, -1, -1)$ and perpendicular to $\mathbf{n}$ is given by

$$5(x-1) + 7(y+1) + 3(z+1) = 0$$
$$\Rightarrow 5x + 7y + 3z + 5 = 0$$

$\therefore$ Required distance

$$= \left| \frac{5 + 21 - 21 + 5}{\sqrt{5^2 + 7^2 + 3^2}} \right|$$

$$= \frac{10}{\sqrt{83}} \text{ units}$$

10 Consider the line through $(2, -1, 3)$ with DC's $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ is

$$\frac{x-2}{2/3} = \frac{y+1}{2/3} = \frac{z-3}{1/3} = r \quad \text{[say]}$$

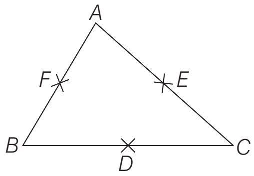
$$\therefore x = 2 + \frac{2r}{3}, y = -1 + \frac{2r}{3}, z = 3 + \frac{r}{3}$$

Since, it lies on the plane $x + 2y - z = 2$.

$$\therefore 2 + \frac{2r}{3} - 2 + \frac{4r}{3} - 3 - \frac{r}{3} = 2$$

$\Rightarrow r = 3$

11 Given, mid-points of sides are $D(l, 0, 0)$, $E(0, m, 0)$ and $F(0, 0, n)$



Also, $EF^2 = \frac{BC^2}{4}$ [by mid-point theorem]

$\Rightarrow BC^2 = 4(m^2 + n^2)$

Similarly, $AB^2 = 4(l^2 + m^2)$

and $CA^2 = 4(l^2 + n^2)$

$$\therefore \frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2} = 8$$

12 $\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

and $\cos^2\alpha = 1 - \sin^2\alpha$, similarly for all other angles.

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$
$= 4 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta)$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma$
$+ \sin^2\delta = \left(4 - \frac{4}{3}\right) = \frac{8}{3}$

13 Let a, b and c be the direction ratios of the required line.

Then, its equation is

$$\frac{x-3}{a} = \frac{y-0}{b} = \frac{z-1}{c} \quad \text{...(i)}$$

Since, Eq. (i) is parallel to the planes $x + 2y + 0z = 0$ and $0x + 3y - z = 0$.

Therefore, normal to the plane is perpendicular to the line.

$\therefore a(1) + b(2) + c(0) = 0$

and $a(0) + b(3) + c(-1) = 0$

On solving these two equations by cross-multiplication, we get

$$\frac{a}{(2)(-1) - (0)(3)} = \frac{b}{(0)(0) - (1)(-1)} = \frac{c}{(1)(3) - (0)(2)}$$

$$\Rightarrow \frac{a}{-2} = \frac{b}{1} = \frac{c}{3} = \lambda \quad \text{[say]}$$

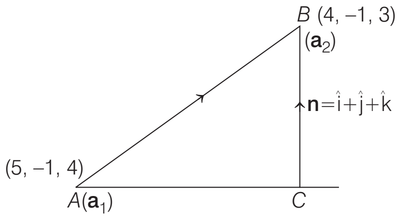
$\Rightarrow a = -2\lambda, b = \lambda$ and $c = 3\lambda$

On substituting the values of a, b and c in Eq. (i), we get the equation of the required line as

$$\frac{x-3}{-2} = \frac{y-0}{1} = \frac{z-1}{3}$$

14 **Key idea** *length of projection of the line segment joining* $\mathbf{a}_1$ *and* $\mathbf{a}_2$ *on the plane* $\mathbf{r} \cdot \mathbf{n} = d$ *is* $\left| \frac{(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{n}}{|\mathbf{n}|} \right|$

Length of projection the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane $x + y + z = 7$ is



$$AC = \left| \frac{(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{n}}{|\mathbf{n}|} \right|$$

$$= \frac{|(-\hat{i} - \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})|}{|i + j + k|}$$

$$AC = \frac{|\hat{i} - \hat{k}|}{\sqrt{3}}$$

$$\Rightarrow AC = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$

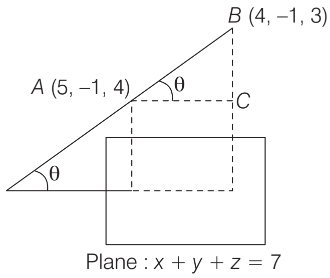
Alternative Method

Clearly, DR's of AB are $4-5, -1+1, 3-4$, i.e. $-1, 0, -1$ and DR's of normal to plane are 1, 1, 1.

Now, let θ be the angle between the line and plane, then θ is given by

$$\sin\theta = \frac{|-1 + 0 - 1|}{\sqrt{(-1)^2 + (-1)^2}\sqrt{1^2 + 1^2 + 1^2}}$$

$$= \frac{2}{\sqrt{2}\sqrt{3}} = \sqrt{\frac{2}{3}}$$



$$\sin\theta = \sqrt{\frac{2}{3}} \Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$= \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$$

Clearly, length of projection

$$= AB\cos\theta = \sqrt{2}\frac{1}{\sqrt{3}} \quad [\because AB = \sqrt{2}]$$

$$= \sqrt{\frac{2}{3}}$$

15 Centroid of ΔABC,

$$G = \left(\frac{2 - 1 + \lambda}{3}, \frac{3 + 3 + 5}{3}, \frac{5 + 2 + \mu}{3} \right)$$

$$= \left(\frac{1 + \lambda}{3}, \frac{11}{3}, \frac{7 + \mu}{3} \right)$$

Since, median is always passes through centroid and they are equally inclined.

$$\therefore \frac{1 + \lambda}{3} - 2 = \frac{11}{3} - 3 = \frac{7 + \mu}{3} - 5$$

$$\Rightarrow \frac{\lambda - 5}{3} = \frac{2}{3} = \frac{\mu - 8}{3}$$

$$\Rightarrow \lambda = 7, \mu = 10$$

16 Let parallel plane be $2x - 2y + z + \lambda = 0$. It passes through $(1, -2, 3)$.

$\therefore \lambda = -9$

The distance of $(-1, 2, 0)$ from the plane $2x - 2y + z - 9 = 0$

is $\left| \frac{-2 - 4 - 9}{3} \right| = 5$

17 Equation of any plane through the intersection of planes can be written as

$$x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y$$
$$+ (1 + 4\lambda)z - 1 - 5\lambda = 0 \quad \text{...(i)}$$

The direction ratios, a_1, b_1, c_1, of the plane are $(1 + 2\lambda), (3\lambda + 1)$ and $(4\lambda + 1)$.

The plane in Eq. (i) is perpendicular to $x - y + z$ = direction ratios, a_2, b_2, c_2 are 1,–1 and 1.

Since, the planes are perpendicular.

$$\therefore \quad a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow 1(1 + 2\lambda) - 1(1 + 3\lambda) + 1(1 + 4\lambda) = 0$$

$$\Rightarrow \quad 1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$$

$$\Rightarrow \quad 3\lambda = -1 \Rightarrow \lambda = -\frac{1}{3}$$

On substituting this value of λ in Eq. (i), we obtain the required plane as

$$\left(1 - \frac{2}{3}\right)x + \left(1 - \frac{3}{3}\right)y + \left(1 - \frac{4}{3}\right)z - 1 + \frac{5}{3} = 0$$

$$\Rightarrow \quad \frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} = 0$$

$$\Rightarrow \quad x - z + 2 = 0$$

This is the required equation of the plane.

18 The two normal vectors are $\mathbf{m} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{n} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

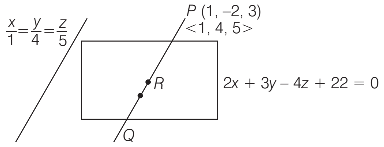
The line L is along, $\mathbf{m} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix}$

$= 3(\mathbf{i} - \mathbf{j} + \mathbf{k})$

and DC's of X-axis are $(1, 0, 0)$.

$$\therefore \quad \cos\alpha = \frac{3(\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot (\mathbf{i})}{\sqrt{3^2(1 + 1 + 1)}\sqrt{1}} = \frac{1}{\sqrt{3}}$$

19 Any line parallel to $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ and passing through $P(1, -2, 3)$ is



$$\frac{x - 1}{1} = \frac{y + 2}{4} = \frac{z - 3}{5} = \lambda \quad \text{(say)}$$

Any point on above line can be written as $(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$.

∴ Coordinates of R are $(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$.

Since, point R lies on the above plane.

$$\therefore \quad 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$$

$$\Rightarrow \quad \lambda = 1$$

So, point R is $(2, 2, 8)$.

Now,

$$PR = \sqrt{(2 - 1)^2 + (2 + 2)^2 + (8 - 3)^2} = \sqrt{42}$$

$$\therefore \quad PQ = 2PR = 2\sqrt{42}$$

20 Here, plane, line and its image are parallel to each other. So, find any point on the normal to the plane from which the image line will be passed and then find equation of image line.

Here, plane and line are parallel to each other. Equation of normal to the plane through the point $(1, 3, 4)$ is

$$\frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = k \quad \text{[say]}$$

Any point on this normal is $(2k + 1, -k + 3, 4 + k)$.

Then, $\left(\frac{2k + 1 + 1}{2}, \frac{3 - k + 3}{2}, \frac{4 + k + 4}{2}\right)$ lies on plane.

$$\Rightarrow \quad 2(k + 1) - \left(\frac{6 - k}{2}\right) + \left(\frac{8 + k}{2}\right) + 3 = 0$$

$$\Rightarrow \quad k = -2$$

Hence, point through which this image pass is

$$(2k + 1, 3 - k, 4 + k)$$

i.e. $[2(-2) + 1, 3 + 2, 4 - 2] = (-3, 5, 2)$

Hence, equation of image line is

$$\frac{x + 3}{3} = \frac{y - 5}{1} = \frac{z - 2}{-5}$$